Applied Game Theory

Polarized Range Examples

No real PLO situations are quantitatively solvable in the way the toy games in the previous sections are. There are some cases that can be close to nuts-or-nothing when the aggressor has a strong enough range, but even these usually involve overlapping ranges, trailing hands with some equity, and/or board texture shifts that change the range versus range dynamic. For example, on monotone boards a significant piece of the range that will continue against someone representing the nuts is sets and the possibility of the board pairing has to be accounted for in assessing multi-street plans. On boards like AAK or KKJ where the nut full house is entirely or nearly invulnerable, the frequency with which people have it is low enough that value betting ranges should usually include lower full houses and sometimes even trips hands all the way through the river. Even the driest rainbow possible-straight and straight-free boards, like T76r and K82r, contain the possibility of backdoor texture shifts.

A second case where real PLO is more complex than game-theoretic analysis would like it to be is river scenarios. Even the toy game scenarios above did not look at full-street games with overlapping ranges, although toy versions of these can be solved if we use enough firepower.[33] Kuhn's Poker is mathematically easier to analyze because there are no thin value-bets or thin bluffs and the nuts-or-nothing game is effectively a **half-street game**, technically defined by Chen and Ankenmann as a game where the first player is forced by the rules to check dark and cannot check-raise.[34] But the core mathematical difficulty with real PLO river situations is not that both players have betting ranges but the effect of card removal on relative hand strengths. The section "The Card Removal Effect" later in this chapter deals with this topic.

The Naked Ace Play

For a first polarized range example, consider the quintessential PLO nuts-or-nothing scenario - a monotone board where one player holds the ace of the suit (and might or might not have a flush). It is strikingly similar to the invented nuts-or-nothing example above, less pure of course but still a closer approximation than nearly any other real poker scenario.

There are three different cases where a naked ace scenario can play out over multiple streets. One, a monotone flop where the flop action does not narrow the participants' ranges significantly. Two, a flop with a flush draw that completes on the turn and where there ws flop action that indicates both/all players have hit the board at least moderately well, putting flush draws as a meaningful part of everyone's range. Three, a monotone flop where the flop action indicates a strong chance that at least one person has a flush.

An example of the first case is a heads-up or three-way pot between reasonably aggressive players. The chance of someone flopping a flush in these cases is only ~35% when heads-up and ~50% when three-ways. Some percentage of these are non-nut flushes that might not bet, usually for pot control reasons. The chance that someone bets the flop without a flush is fairly high, especially in the heads-up case where the preflop raiser's continuation bet frequency could be as high as 70%-80%. The result is that even a tight bettor usually cannot establish that he is representing the nuts until he bets the turn; to the extent that these are nuts-or-nothing scenarios, they play out over two streets. Similarly, when there is a bet and a call on a suited flop, both nut- and non-nut flush draws are a meaningful portion of people's ranges. But is not until the flush completes on the turn and someone chooses to bet again that he (mostly) narrows his range to a polarized mix of strong flushes and represented flushes.

In each case, Table 6.9b provides a reasonable picture of the game theory behind the turn-river scenario. In the extreme case, if Player A is known to have the nut card (and possibly a second of the suit), Player B has a bluff-catcher, and Player A properly balances his turn and river bluff frequencies, then Player B cannot do better than folding as long as A has the nuts more than 44.4% of the time. This sounds like an extremely high frequency, but it occurs in practice more than one might realize.

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[33] See <u>Mathematics of Poker</u> for examples.

[34] In the nuts-or-nothing game, Player A is forced to check dark by the complete information asymmetry.